This is one of the two best engineering whitepapers that I know of, about the complex issues of high-speed-digital paging. It was written by my friend and colleague, Allan Angus, PE. (He patiently taught me a lot about Paging.). PageMart Wireless later became WebLink Wireless, and then was acquired by Metrocall. Arch and Metrocall merged, and became USA Mobility.

The companion paper to this one is: "Pitfalls on the way to high speed paging from the service provider's perspective", by *Selwyn E. Hill, who was a* senior RF engineer at PageMart/WebLink.

PageMart Wireless

FLEX at 6400 bit/s

Version 1.0
December 15, 1997
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Table of Contents

1. Summary

This report reviews the FLEX protocol and suggests new causes for the observed problems with operations at 6400 bit/s. It is necessarily technical in content. To the greatest extent, care has been taken to describe the technical material in a manner that is accessible to readers familiar with paging. Unfortunately, much of this material is rooted in the mathematics of electrical engineering and random variables. To the uninitiated, a note of caution: proceed at your own risk.

The report offers some conclusions for the problems with FLEX at 6400 bit/s; namely, that old bug bear, FM click noise. In traditional land-mobile communications, FM click noise is associated with noise capture in limiter-discriminator receivers. In simulcast paging, it transpires that there are two distinct situations than guarantee the generation of clicks. These have to do with simulcast beat fading and with simulcast delay spread.

It is also shown that a typical FLEX receiver will be uniquely impacted by click noise, where earlier binary POCSAG pagers would not. A new explanation of why ReFLEX25 devices show superior performance to FLEX is also presented.

The report includes a few recommendations. First, a review of the probability statistics for signal strength in simulcast situations is over-due. Second, the implementation of frequency offset plans in specified market scenarios is called for. Third, further exploration of new algorithms for simulation and optimization of delay spread effects is needed. Finally, receiver designs that avoid click noise are described, and the device vendors should be encouraged to adopt improved designs for high-speed operation.

2. FLEX Protocol

The FLEX protocol supports one-way paging applications at UHF mobile frequencies. Unlike the earlier Post Office Code Standardization Activities Group (POCSAG) protocol, FLEX is synchronous. Like POCSAG, the FLEX transmission is based on a simple Frequency Shift Keying (FSK) modulation.

FLEX may be transmitted at a number of different bit rates and baud rates. This can lead to confusion as to its basic structure. No matter the transmission rate, any FLEX receiver ultimately decodes a 1600 bit/s stream. The other possible bit rates, 3200 bit/s and 6400 bit/s, are comprised of aggregates of two or four of these basic streams. Another confusing aspect of FLEX is that this aggregation, or "multiplexing" is performed right at the bit level, or physical layer, of the protocol.

FLEX also embodies the notion of "phase" in its signaling. Pagers are assigned one of four "phases", called A, B, C, or D. Which of these four phases are relevant depends upon the signaling speed. Generally, a FLEX pager receives a bit stream at 1600 bit/s independent of the channel speed. This implies a channel time division multiplex method. In FLEX, this is achieved at the symbol level.

At 1600 bit/s, phasing is not important. At 3200 bit/s, with binary signaling, alternate bits are assigned to the A phase and B phase. At 3200 bit/s, with quaternary signaling, the most significant bit (MSB) of a symbol is assigned to the A phase, while the least significant bit (LSB) is assigned to the B phase. At 6400 bit/s, alternate symbols are assigned to the A and B phases or the C and D phases. Within a symbol, the MSB is assigned to the A phase (or C phase for the next symbol) and the LSB is assigned to the B phase (or D phase).

This leads to a very different error process for A and C phase signaling than for B and D phase signaling whenever quaternary signaling is used, as will be seen below.

2.1 Modulation

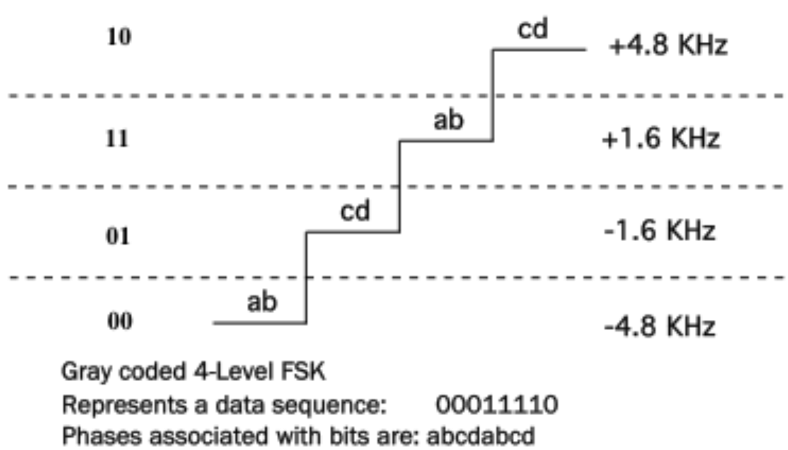
The FLEX modulation depends upon the bit rate and baud rate. It is either binary or quaternary (4-level) FSK. Binary modulation is used at 1600 bit/s and optionally at 3200 bit/s. Quaternary FSK is used optionally at 3200 bit/s and at 6400 bit/s. The peak frequency deviation in all cases is 4800 Hz. For 4-level modulations, the inner symbols use a frequency deviation of ±1600 Hz. Thus, the baud rates are either 1600 baud or 3200 baud.

In FLEX binary signaling, ‘0’ corresponds to -4800 Hz and ‘1’ to +4800 Hz. In FLEX quaternary signaling, there is a “Gray code” mapping according to this table.

10	+4800 Hz
11	+1600 Hz
01	-1600 Hz
00	-4800 Hz

The following figure shows an example of a 4-level, 4-“phase” transmission.

Figure 2-1 The 4 phases of FLEX at 6400 bit/s



Conceptually, the modulation is developed by passing Non-Return to Zero (NRZ) Pulse Amplitude Modulated (PAM) square waves through a 10th order low pass Bessel filter with a 3 dB point of 3.9 kHz and sending the resultant symbol stream to a frequency modulator at the carrier frequency. The same low pass break point is used for both baud rates. The Bessel filter is used for its linear phase characteristics, which is desirable in data transmission applications.

If the PAM levels are assumed to be ±1 V and ±3 V, then the frequency modulator can be assumed to be a voltage controlled oscillator (VCO) with a conversion factor of 1600 Hz V^{-1}.

The FLEX protocol has a complex higher layer architecture as well, but little of that is relevant to the present discussion. Interested readers are referred to the most recent Motorola protocol document.

2.2 Reception

Reception of the FLEX symbol stream may be done in a number of ways. For economical paging receivers, the structure generally includes a first stage RF amplifier (often with automatic gain control or AGC), a mixer, a first intermediate frequency (IF) filter and amplifier, another mixer and second IF filter and limiter amplifier, a frequency discriminator, a base-band receive filter, automatic level control, symbol clock recovery, and a 4-level slicer (2-bit ADC).

Variations on this design may skip the first stage RF gain block (to avoid intermodulation distortion problems), and one or more of the IF conversions[1]. These variations are not particularly relevant to this analysis. The critical elements will turn out to be the pass-band IF filter, the frequency discriminator, the receive filter, and the automatic level control.

3. UHF Paging Mobile Radio Channel

The UHF mobile radio channel has been subject to an enormous amount of study of the past few decades. Unfortunately, much of the attention has been focused on single transmitter land mobile applications such as broadcast FM, trunked radio, cellular voice, and cellular data. Paging is characterized by the use of simulcasting transmitters to achieve higher average levels of signal power than are typical of, cellular radio, for example.

This is a fundamental distinction between paging and cellular (or wide-band PCS). In cellular radio, high system capacity is achieved by the densest possible re-use of the available frequency. This is obtained through modulations and protocols that are highly immune to co-channel interference. A typical digital cellular system, IS-136 or "TDMA," operates at a design threshold of 3% bit error rate (BER) at carrier to interference ratios (CIR) of about 13 - 14 dB. In contrast, a typical paging receiver may operate at a design threshold of 99% message reliability at a received signal strength of -88 dBm. The theoretical noise power in a 50W resistor at room temperature is about 1 nV Hz-½. Assuming a noise bandwidth of 25 kHz and a noise figure (NF) of about 3 dB, the noise power is about -120 dBm. So the paging receiver operates at a carrier to noise ratio (CNR) of about 32 dB.

The difference of nearly 20 dB in CIR and CNR, even in this simplistic example, is indicative of a major difference in the operating modes of cellular and paging. As a consequence of this strong difference, higher layers of protocols such as IS-136 and the FLEX family show similar distinctions in their methods of error coding. Again, while the fine details of these differences are not pertinent to this discussion, it is important to recognize that any process that impacts the probabilities of signal or noise, or alters the error mechanisms, from the more extreme design values needed for the FLEX family will be of concern.

In the following, it will transpire that the relevant processes include two that are well-known in single transmitter land mobile; namely, fast Rayleigh fading and slow log-normal shadowing, and two that are unique to simulcast; namely, simulcast beat fading (SBF) and simulcast delay spread (SDS.) FLEX at 6400 bit/s is particularly impacted by error processes related to both SBF and SDS effects in the channel.

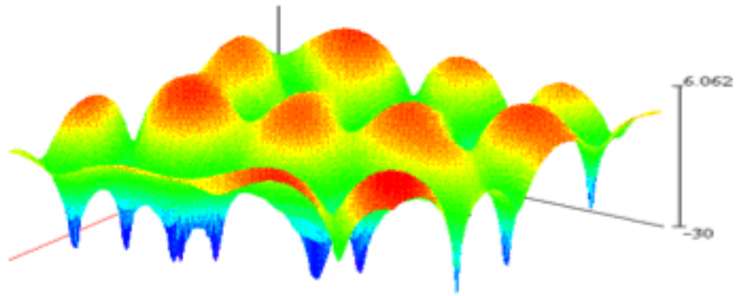
There are two other channel processes, which are due to terrain-based signal scattering, that are of relevance to wide-band communications. These are random Doppler (RD) and excess delay spread (EDS.) RD remains of interest in paging; EDS is typically a much lower-order effect than SDS, and can generally be neglected.

3.1 Rayleigh Fading

If there were only one path that radio waves could take between a transmitter and receiver, then far fewer RF engineers would be employed, and their jobs would be much easier. As luck would have it (for RF engineers), radio waves can be reflected off surfaces; they can refract (or bend) around the corners of objects; and they can diffract (or interfere constructively and destructively in complex ways) through arrays of obstacles and apertures. These three processes are typically lumped together under the name, "scattering." The opportunities for reflection, refraction, and diffraction depend upon the wavelength and the scale of objects in the environment. At paging frequencies (around 901-940 MHz) wavelengths are about 30 cm. This implies that there are many objects in the environment (from the earth itself to tree leaves) that can scatter paging radio signals.

Imagine a single, un-modulated carrier being transmitted from a fixed antenna. If one assumes that scatterers are uniformly and randomly distributed over the earth's surface, and that the signal strengths from all these sources are random and obey the gaussian probability distribution, then the interference of all of the scattering sources (together with the fixed antenna itself) creates a singular pattern of deep signal nulls and broad, local peaks in space. This is shown in the following figure.

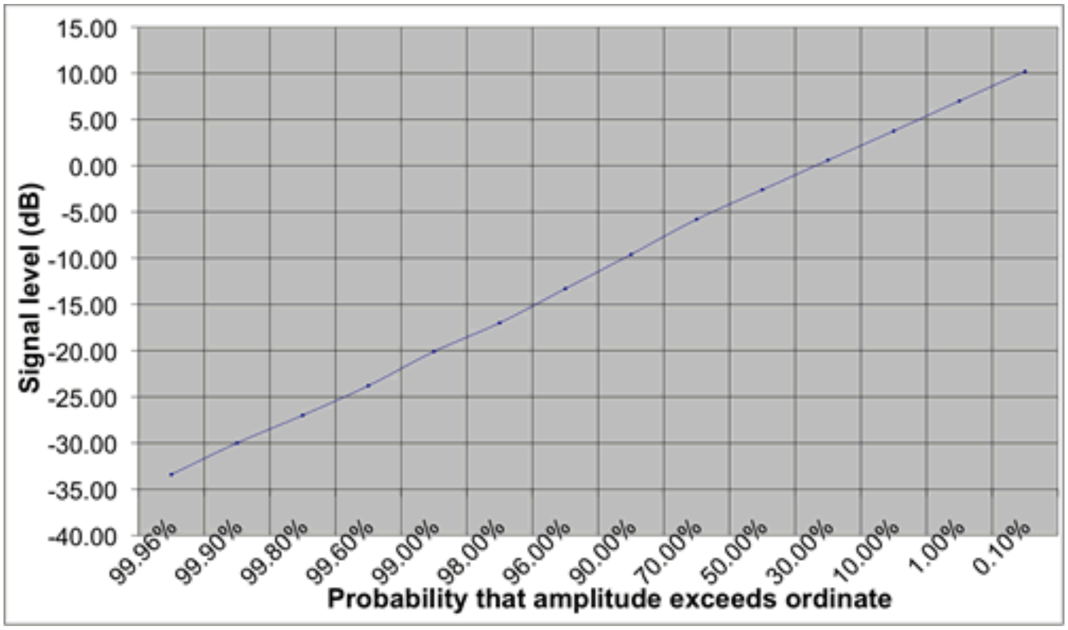
Figure 3-1 Rayleigh fading pattern in 3-D



If the scatterers were stationary, then this pattern would be frozen in time. In practice, there is always some small degree of motion in the environment; and the pattern of nulls and peaks shifts slowly and randomly. Some things can be said about the pattern, however. First, the deep nulls are spaced about ½ wavelength apart. Second, the phase of the received signal is roughly constant in the space between nulls (that is, under the broad, local peaks), but shifts to another random value under the next broad peak.

Taken as a whole, the distribution of signal strengths follows the so-called "Rayleigh" distribution (see Figure 3-2), which is derived by considering the square root of the sum of two squared Gaussian distributions[2]. When a mobile receiver is moved rapidly through this quasi-stationary pattern of nulls and local peaks, its antenna detects a fading signal. The rate of fading is about ½ wavelength divided by receiver speed. At paging frequencies, ½ wavelength is around 30 cm. For example, with a carrier of 900 MHz and at a speed of 30 mph (13.3 m s^{-1}), the fade rate is 40 s^{-1} (40 Hz.) If the receiver is stationary and the environment of scatterers is subject to a slow random motion of about 3 mph (1.33 m s^{-1}), then the fade rate is only 4 Hz.

Figure 3-2 Rayleigh Cumulative Probability Distribution



In short, the Rayleigh distribution may accurately describe the received signal strength available to a mobile receiver; but the dynamics of how the receiver samples that distribution depends heavily on its motion.

When the mobile receiver is at ground level, the impact of signal scattering will be maximized. As the receiver is elevated from ground level, a point is achieved at which a clear line of site is available to the transmitter. Rice has developed a PDF for signal strength in situations like this in which a strong line-of-sight path is added to a Rayleigh background. In a receiver location at good elevation, for example, an office tower, a Ricean PDF is a better model for signal strength. This observation will be important in the discussion of simulcast beat fading (3.5.)

3.2 Random Doppler

If a radio receiver moves directly towards a transmitter with a carrier at a frequency of f_c then it detects a frequency at $f_c + f_D$, where $f_D = f_c\,(v/c)$; v is receiver velocity and c is the speed of light. This frequency offset is called a Doppler shift, and in this case it is deterministic. If the receiver is moving directly away from the transmitter, the shift is below the carrier by $-f_D$. More generally, if there is an angle Θ, between the vector of the inbound radio wave and the receiver's velocity, then the frequency shift is $f_D\cos(\Theta)$.

Now, continue with the scenario described in the previous section. If the paging receiver moves through the RF field generated by random scattering, the detected carrier is subject to a random Doppler shift. The typical method of analysis integrates the gaussian distribution of in-phase and quadrature components over all 2π radians between radio wave numbers (or vectors) and receiver velocity. The result is a double-peaked power spectral density (PSD) curve, $S(f)$, that tends to infinity at $\pm f_D$. That is,

$$S(f) = \frac{C}{\sqrt{1-\left(\frac{f-f_c}{f_D}\right)^2}},$$

where C is a constant that depends on signal strength.

Random Doppler is a frequency dispersion process that affects all signals moving through the mobile radio channel. In the case just analyzed, there was only a single (monochromatic) carrier. In the frequency domain, the process is one of convolution of the transmitted signal with the random Doppler PSD (RD-PSD.) In the time domain, the process is multiplication (or modulation) with a random Doppler signal. When a modulated carrier is transmitted through the channel, the received signal is the convolution of the transmitted information (for example, the four distinct FSK symbols of 6400 bit/s FLEX) and the RD-PSD. Each symbol is subject to frequency dispersion (or spreading) equal to twice the Doppler spread.

In most cases, paging FSK symbols are subject to a broader spread by the pulse shaping filters used in the transmitter and receiver. The greatest effect of RD in the channel is to produce timing jitter in the recovered symbol clocks, and in detected carriers in synchronous receivers[3]. In this way, RD establishes an "irreducible bit error rate" floor, which depends upon vehicle velocity, and cannot be overcome by any value of increased signal strength.

This effect is of pragmatic importance in the interpretation of BER data acquired during drive tests. At precisely the same location, different BERs will result depending upon vehicle velocity. At low speeds, where random Doppler dispersion is unimportant, BER will depend mainly upon signal strength (and other effects to be described below.) At high speeds, the irreducible BER of RD will come into play; and areas of drive testing (say, at freeway speeds) may show unusually poor paging in contrast to near-by regions (tested at lower speeds.)

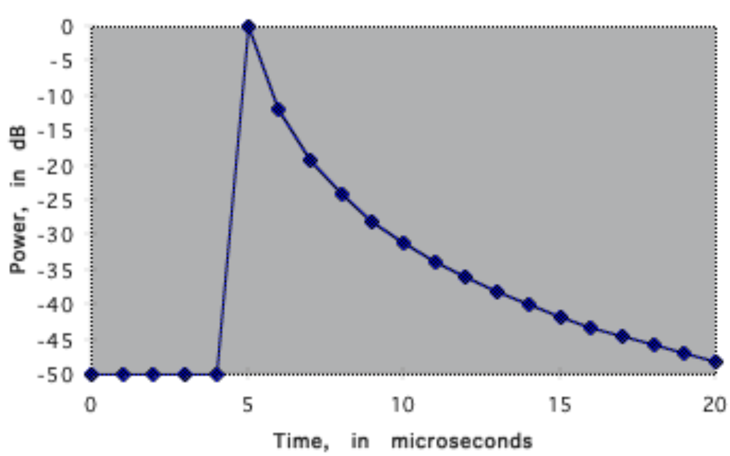
"Quasi-random" Doppler will also occur in a simulcast environment without any scattering process at all. Consider an urban paging situation in which 10 or more simulcast transmitters are visible to a mobile paging receiver. Assume that the direct paths from each transmitter to the mobile are at least approximately random; that is, the angles their vector wave numbers[4] make with respect to the mobile velocity vector are about uniformly distributed over *2π* radians. The resulting PSD will have distinct peaks at the resulting Doppler shifts of all of the individual carriers, and the over-all PSD will be under an "envelope" characteristic of RD.

Below, we will consider the impacts of a frequency offset plan in a simulcast environment. It may be noted here that such offset plans will generally increase the effect of Doppler dispersion and will also tend to increase the irreducible BER at any given value of mobile receiver speed. Having said that, the benefits of offset plans out-weigh their costs for FLEX at 6400 bit/s.

3.3 Excess Delay Spread

Let us return to the same scattering environment that we have considered in the past two sections; but now instead of transmitting a pure carrier, let us send a (infinitesimally) short burst of RF[5]. The phenomenon we now see is a dispersion of power over a range of times (instead of frequencies) due to the distinct lengths of the various scattering paths (and the constant speed of light.) In general, there will be some shortest delay along the most direct path. Also, since radio signals are attenuated in inverse proportion to some power law of path length[6], a fairly typical distribution of power looks like the figure below.

Figure 3-3 Power dispersion due to excess delay spread



The minimum delay spread in this figure is 5 μs, corresponding to a minimum path length of 1.5

km. The curve shows an "excess delay spread" of power due to scattering along paths longer than the minimum of 1.5 km.

It is straightforward to define a power-weighted mean delay, <τ>, as

$$\langle \tau \rangle = \frac{\int_0^\infty P(t) \cdot t \cdot dt}{\int_0^\infty P(t) \cdot dt}.$$

It is then natural to define a root mean square (rms) measure of dispersion as a power-weighted standard deviation. This measure is called the rms excess delay spread (EDS), τ_m. It is defined as

$$\tau_m = \frac{\int_0^\infty P(t) \cdot \left(t - \langle \tau \rangle\right)^2 \cdot dt}{\int_0^\infty P(t) \cdot dt}.$$

As an example, for the curve in the figure above, the mean delay is about 5.1 µs, and the rms excess delay is about 0.23 µs. More practically, rms EDS values range from around 2 or 3 µs in flat rural areas to 10 or 12 µs in urban environments to over 100 µs in extremely hilly and mountainous terrain.

For linear modulations, the impact of EDS in the channel is to generate multiple, time-offset copies of the transmitted information stream at the receiver. This "inter-symbol interference" (ISI) creates an irreducible bit error rate effect similar to that caused by RD. That is, there will be an "error rate floor," which depends only on the magnitude of τ_m, and which no value of increased signal strength can improve.

For linear modulations, there is a "rule of thumb" for the threshold at which the ISI due to EDS becomes significant. The rule is that τ_m should be less than 25% of a symbol time (or baud.) In the case of FLEX at 6400 bit/s, a baud is 1/3200 Hz = 313 µs; and the threshold of ISI is about 78 µs. The following table shows the relationships between channel speed, symbol time, the ¼ symbol time ISI threshold, and the distance that radio signals will travel during the ISI threshold.

Speed (baud)	**Symbol time (µs)**	**ISI limit time (µs)**	**ISI distance (mi.)**
512	1953	488	91.55
1200	833	208	39.06
2400	417	104	19.53
1600	625	156	29.30
3200	312	78	14.65

First, it can be seen how the ISI distance collapses as the signaling speed is increased. However, in the absence of fading or shadowing of radio signals, it would be difficult to create situations in which the ISI distance criterion was exceeded. Over a smooth earth, signals fall off at about the inverse fourth power of distance. To have a situation with bad ISI, one would have to have a receive location 15 miles closer to one site than another, yet the signal from the more distant site would remain about equal to that from the closer site. That is impossible to get, without fading or shadowing. For an example, assume that the receive location is 7.5 miles from one site and 22.5 miles from the other. The relative strengths of the two signals should be $10*log(3^4) = 20$ *dB* different. As the distance between the receive location and the two sites is increased, the relative attenuation over the 15 mile difference becomes smaller. At a distance of about 35 miles from the closest site, the relative attenuation is about 6 dB. One has to go to 80 miles out to get a 3 dB difference.

Let us make two observations for now. First, the EDS due to most scattering environments is insufficient to create ISI problems for FLEX, even at the highest baud rates. Second, because FSK is a non-linear modulation, the dominant effect of delay spread will turn out *not* to be ISI at all. (See section 3.6.)

3.4 Log-Normal Shadowing

Because of its motion, mobile receiver frequently goes through scenarios in which a large object, such as a building or hill, comes between it and the transmitter. As the receiver passes into the "RF shadow" of the obstruction, it experiences a deep and rapid fade in signal strength. Consider a situation in which a mobile receiver very slowly followed a circular path around a fixed omni-directional transmitter antenna sending an un-modulated UHF carrier. We want the receiver motion to be slow, so that it's average detected signal smoothes out any fast Rayleigh fading effects. Because of random obstructions along the path between the transmitter and receiver, the local average signal strengths will still be randomly distributed.

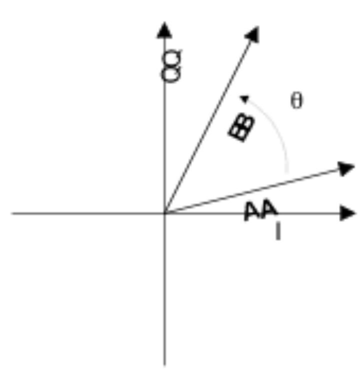
In general, this distribution turns out to be approximately Gaussian (or Normal) on logarithmic (or dB) scales. Hence, this effect is called "log-normal shadowing." In flat, rural environments, the standard deviation, σ, of the log-normal curve is typically 5 dB or so. In heavily built-up urban environments, σ can be as high as 12 dB or more. It is customary to use values of σ around 6 to 8 dB.

From the point of view of an RF engineer attempting to ensure that RSSI exceeds some threshold of paging reliability, the issue of an accurate determination of s is obvious. Let us say that a simulation program predicts that mean RSSI will be -70 dBm at some location, and that -88 dBm is necessary for adequate paging. We have an 18 dB margin. However, if σ is 6 dB, then there is only a 0.13% probability due to log-normal shadowing[7] that the receiver will see a mean signal level lower than -88 dBm. On the other hand, if σ is 12 dB[8], the probability of a level lower than the reliability threshold is 6.7%. In the first case, paging reliability is degraded to no more than 99.97% by shadowing; in the second, the reliability has fallen to 93.3%. The difference is clear.

3.5 Simulcast Fades

Consider two signals, **A** and **B**, at exactly the same carrier frequency, f_c. The phasor diagram below shows the two signals, with an assumed constant phase difference of Θ.

Figure 3-4 Phasor diagram for simulcast signals



If the two signals have some small frequency offset, say, ω_{off}, then we can imagine the phasor, **B**, rotating around **A**. In other words, $\Theta = \omega_{off}t$. If **B** is nearly equal **A** in amplitude, then whenever **B** and **A** are exactly out of phase ($\Theta = \pi$ radians), the resultant phasor sum will be a minimum. If B and A are equal in amplitude, their sum will be 0 at these instants of destructive interference.

In practice, all simulcast transmitters will have some frequency offset. In any receiving location where signals from two dominant transmitters are received, a deterministic sequence of fades will occur at the "beat frequency" due to these natural offsets.

In POCSAG systems, *engineered* frequency offsets have always been recommended to manage this effect. If the beat frequencies are small, of the order of only a few Hertz, a paging receiver at a fade location can be there for a significant time. Engineers have recognized that received data will be corrupted if the signal falls below receiver sensitivity.

In an engineered offset plan, each transmitter's carrier frequency is purposely shifted from the nominal channel center. There are many ways to do this. This simplest to deal with picks a number of offsets, and then imposes them on the transmitters in a manner similar to a cellular re-use plan. In this way, no neighboring transmitters have the same offsets.

A typical frequency offset plan for POCSAG has offsets in a range of ±500 Hz. Most POCSAG pagers are designed for 4500 Hz deviation. Hence, an offset of an additional 500 Hz would not exceed the band limit of 5 kHz and a deviation of 4000 Hz could easily be accommodated by the paging receiver. Before the advent of FLEX, transmitters were not required to have the frequency stability needed for 4-level modulation, and most transmitters had an inherent, random offset because of carrier frequency drift. These random drifts may not have been as much as 500 Hz, but there was probably enough difference between transmitters to essentially do the job without intervention. Paging reliability at lower baud rates was good enough. PageMart engineers did not implement a POCSAG frequency offset plan.

For FLEX, vendors specifically advised against an engineered offset plan. The FLEX protocol expects modulation accuracy on the order of ±10 Hz. It appeared that offset plans of the scale used for POCSAG would have disastrous effects on FLEX.

Consider for a moment the joint impact of the constructive and destructive interference of simulcast upon the probability distribution of signal strength in an environment that is already subject to Log-normal fading. Constructive interference will allow the addition of peak signal powers. Destructive interference will allow for fades to *0* signal strength. On logarithmic (dB) scales, the Log-normal curve will become asymmetric with a long tail on the negative side.

This *possible* change in the PDF of signal strength in simulcast environments is of significant importance in the calculation of signal margins for paging reliability. **It should be the subject of further experimentation.**

One more arcane aspect of RF engineering should be observed in this context. It may be recalled that signals are subject to fast Rayleigh fading in a mobile environment. Our analysis here has assumed that signal strength is a constant; and this assumption does not fit the model of signals strongly impacted by Rayleigh fading.

However, simulcast fading remains an issue in Ricean fading environments; that is, in situations where a clear line of sight is available to two or more transmitters. The situations in which this occurs will be in-doors in office towers and out-doors on hilltops. So, it may turn out that any modification to the PDF of slow (Log-normal) fading happens only in situations where the fast fading is Ricean.

3.6 Simulcast Delay Spread

We have previously mentioned the situation of excess delay spread due to terrain factors. In a simulcast environment, excess delay exists simply due to the different path lengths between the various transmitters and the paging receiver.

Like the measure of rms EDS, a measure of SDS has been proposed. This measure, from Hess, is the multipath spread, T_m, which is just twice the RMS excess delay. The concept underlying this measure is simple: any arbitrary value of RMS excess delay can be achieved by spacing two signals of equal power by a time $T_m = 2\ \tau_{rms}$.

Hess' measure for multipath spread for N simulcast signals is given by

$$T_m = 2\sqrt{\frac{\sum_{i=1}^{N} P_i d_i^2}{\sum_{i=1}^{N} P_i} - \frac{\left(\sum_{i=1}^{N} P_i d_i\right)^2}{\left(\sum_{i=1}^{N} P_i\right)^2}}$$

where P_i and d_i is the power and delay of the *i*-th signal, respectively. The term with the square root is just τ_{rms}. In other words, T_m is just double the τ_{rms}.

An interpretation of T_m is that it is the time difference between two equal-power signals that has the same τ_{rms} as the *N* transmitters do[9].

As mentioned above, it has been assumed that the primary effect of SDS at the paging receiver is to cause inter-symbol interference between adjacent symbols. This assumption is associated with the "¼ symbol time" rule for ISI. In what follows we show that there is another effect, associated instead with signal fades due to a destructive interference process similar to that of the previous section.

Consider again the phasor diagram of Figure 3-4. Imagine now that we have a time delay between **A** and **B**, and that the signal frequency at the beginning of the situation is the FSK modulated carrier at $\omega_0+\omega_1$. Now assume that the symbol changes, and that both **A** and **B** begin to shift to $\omega_0+\omega_2$. Suppose that, because of the relative time delay in the signal paths, the receiver sees the signal from **B** begin to ramp to up first, before **A** does. During this inter-symbol ramp-up time, the phase angle of **B** will shift rapidly against the phase angle of **A**.

We can quantify this process. The carrier frequency is ω_0, the first symbol is ω_1, and the second symbol is ω_2, and the inter-symbol transition time is *T*. We can write an expression for the signal during this period as

$$\vec{C} = \vec{A} + \vec{B},$$

$$If,$$

$$\vec{A} = A\angle(\omega_0 + \omega_1)t$$

$$and$$

$$\vec{B} = B\angle(\omega_0 + \omega_1)t + \frac{(\omega_2 - \omega_1)}{T}t^2$$

$$Then,$$

$$C^2 = \left[A\cos(\omega_0 + \omega_1)t + B\cos\left((\omega_0 + \omega_1)t + \frac{(\omega_2 - \omega_1)}{T}t^2\right)\right]^2 +$$

$$\left[A\sin(\omega_0 + \omega_1)t + B\sin\left((\omega_0 + \omega_1)t + \frac{(\omega_2 - \omega_1)}{T}t^2\right)\right]^2$$

$$and$$

$$\arg(C) = \arctan\left(\frac{\left[A\sin(\omega_0 + \omega_1)t + B\sin\left((\omega_0 + \omega_1)t + \frac{(\omega_2 - \omega_1)}{T}t^2\right)\right]}{\left[A\cos(\omega_0 + \omega_1)t + B\cos\left((\omega_0 + \omega_1)t + \frac{(\omega_2 - \omega_1)}{T}t^2\right)\right]}\right)$$

The arrow over a variable indicates that it is a phasor. In the absence of the arrow, the phasor magnitude is meant. The function *arg(.)* takes the angle of the phasor.

Note that these equations are only valid for $0 \le t \le T$. This shows that the resulting signal changes at ½ the desired rate and is modulated by an envelope that "chirps" at ½ the difference between the two symbols. We can simplify the situation if we take the angle, $(\omega_0 + \omega_1)t$, as a reference, so that **A** is perfectly "in-phase." Then

$$C^2 = \left[A + B\cos\left(\frac{(\omega_2 - \omega_1)}{T}t^2\right)\right]^2 + B^2\sin^2\left(\frac{(\omega_2 - \omega_1)}{T}t^2\right)$$

and

$$\arg(C) = \arctan\left(\frac{\left[B\sin\left(\frac{(\omega_2 - \omega_1)}{T}t^2\right)\right]}{\left[A + B\cos\left(\frac{(\omega_2 - \omega_1)}{T}t^2\right)\right]}\right)$$

We are looking for situations in which destructive interference can occur during the symbol transition; that is, for **C** to become a minimum. As in the case of simulcast beating, this can happen when **A** and **B** are out of phase, which occurs when the argument of the cosine in the above equation is exactly π radians. In this case, **A** and **B** cancel, and the quadrature component is 0.

The question is, how many radians of arc will the phasor **C** go through in a typical transition time? The answer is $2\pi(f_2 - f_1)T$. In FLEX, the possible values of $(f_2 - f_1)$ are 9600, 6400, 3200, and 0 Hz. At 6400 bit/s, the transition time may be 30 µs, assuming that a transition happens in about 10% of a symbol time. The possible fractions of 2π radians are then 28%, 19%, 10% and 0%, respectively.

In practice, the symbol transition will begin with the resultant phasor, **C**, at an arbitrary phase angle. Hence, the probability that an event of destructive interference occurs during the symbol transition is just the fraction of 2π radians that are covered by the resultant during the transition. The fractions that were computed in the last paragraph apply only to ½ of the transition time in the worst case scenario. That is, we looked at the transition for **B** shifting to the new symbol first. To get the whole process, we also need to account for the time that **B** is at the new symbol before **A** begins to shift and the transition of **A** to the new symbol later.

If **B** is stable at the new symbol for some period before **A** starts to shift, then the situation is one of a beat frequency of $(f_2 - f_1)$ between two carriers. The phase angle of the resultant will accumulate at the rate of $2\pi(f_2 - f_1)t$ for the duration of this time. Finally, the process of **A** shifting to the new symbol will be identical to the process of **B** shifting first.

We can summarize the process as follows. As **B** begins to shift, the phase angle of the resultant accelerates to a velocity equal to $2\pi(f_2 - f_1)$. It then holds at this velocity for the duration of time until **A** begins to change. It then decelerates back to *0* once **A** completes its change and the two signals are at the new symbol.

The resultant phasor will be subject to destructive interference for each time the accumulated phase angle passes through *π* radians. Clearly, for low values of SDS, the probability of a destructive interference event occurring during a transition is low. As SDS is increased, it becomes certain that at least one event occurs. This threshold will be different for the different inter-symbol transitions because of the different rates of accumulating phase.

It possible to calculate the value of T_m that will guarantee a destructive interference event for all possible symbol transitions. The values of time turn out to be 100, 150, and 300 µs, respectively, for the 9600, 6400, and 3200 Hz inter-symbol distances. In practice, by averaging over all possible inter-symbol transitions, one arrives at a weighted average of 250 µs to guarantee, on average that there is a destructive interference event during transitions.

This is an important parameter. We shall return to it later in the discussion of click noise that is generated because of destructive fades.

4. Frequency Modulation

Frequency modulation (FM) is a "non-linear" (or angle) modulation in which the signal *s(t)* is

$$s(t) = A\cos\left(2\pi f_c t + 2\pi f_d \int_0^t m(\tau)d\tau\right).$$

If the modulation, *m(t)*, were a constant, 1, then the integral becomes *t*, and *s(t)* experiences a carrier shift of exactly f_d the peak frequency deviation. For analog modulations, it is customary to define a modulation index, β, as

$$\beta = \frac{f_d}{W},$$

where *W* is the maximum frequency content of the base-band signal.

For digital angle modulations (FSK & PSK), it is customary to define a modulation index, β, in a somewhat different manner; namely,

$$h = 2\cdot f_d \cdot T_s,$$

where, by definition, a system with $h=1$ will shift phase at a peak rate of π radians in each symbol time, T_s.

For binary FLEX, f_d is easy to discover, it is just 4800 Hz. Thus, h=2(4800)/1600 = 6 for 1600 bit/s, and h=3 for 3200 bit/s binary signaling. In the quaternary case, f_d is a little trickier. In this case, f_d is actually 1600 Hz, which is the deviation used for the two "inner" symbols. So, h=2 for 3200 bit/s and h=1 for 6400 bit/s quaternary signaling. For integral values of the modulation index, sometimes called the frequency deviation ratio, the transmitted spectrum shows distinct "spurs."[10] Also, for values of $h \geq 1$, the modulation is orthogonal, although not a compact as it can be. Continuous phase FSK with h=0.5 and NRZ transmission pulses is called minimum shift keying (MSK). Gaussian MSK (GMSK), also with h=0.5, uses Gaussian pulses instead of NRZ. GMSK is employed in the GSM system. FLEX signaling occupies much more bandwidth than MSK or GMSK would require for the same symbol rates.

4.1 Noise Capture and Signal Capture

Let the signal at the discriminator be

$$\begin{aligned} e(t) &= Q\cos\omega_c t + n(t) \\ &= [Q + X_c(t)]\cos\omega_c t - X_s \sin\omega_c t \\ &= R(t)\cos[\omega_c t + \theta(t)] \end{aligned}$$

where *Q* is the amplitude of the carrier,ω_c is the carrier radian frequency, *n(t)* is additive noise with in-phase component $X_c(t)$ and quadrature component $X_s(t)$. The amplitude and phase of the signal are:

$$R(t) = \sqrt{[Q^2 + X_c(t)]^2 + X_s^2(t)},$$

$$\theta(t) = \arctan\left[\frac{X_s(t)}{Q + X_c(t)}\right]$$

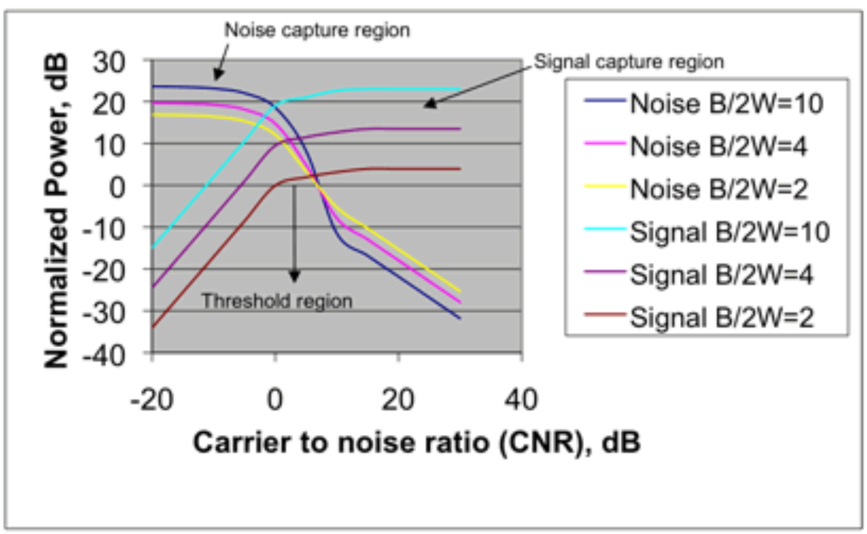
At high values of CNR, the detected frequency is just

$$\theta'(t) = \frac{X_s'(t)}{Q}$$

The primes denote differentiation with respect to time. Note that this applies to the un-modulated carrier; that is, this is just the noise. Since the power spectrum of $X_s(t)$ will correspond to white noise shaped by the IF filter, the PSD of its time derivative will have a quadratic frequency characteristic. Note also that the amplitude of the noise will be suppressed by the inverse of Q^2. That is, increasing signal strength suppresses FM noise.

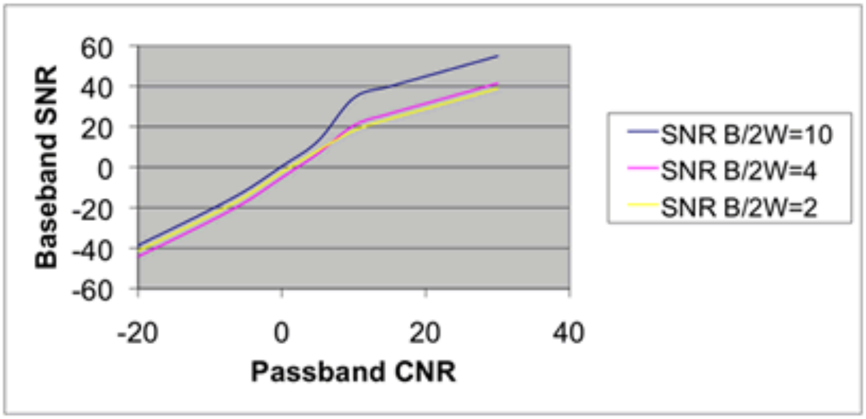
As CNR falls, the noise increases dramatically; and its spectrum shifts to that of the unsuppressed IF filter characteristic. This effect depends on the ratio of pass-band width, *B*, to base-band width, *W*. For CNRs below 0 dB, the output signal strength falls off while the discriminator captures noise. For high CNRs, the signal level saturates and noise is suppressed. Figure 4-1 shows these effects for various ratios of *B/2W*.

Figure 4-1 Noise quieting & signal suppression in FM



Using the same data, Figure 4-2 shows the relationship between base-band SNR to pass-band CNR, given that the FM signal capture effect exists.

Figure 4-2 SNR vs. CNR in FM with capture



It can be seen that there is a significant gain in SNR, if pass-band width can be consumed. In FLEX, the pass-band width depends on the peak frequency deviation, f_d, 4800 Hz, and the symbol rate. The modulation is generated by taking a NRZ square wave at the symbol rate and passing this through a 10th order Bessel filter with a 3 dB cut-off of 3.9 kHz, before FM transmission. The PSD for the NRZ data sequence will have a sinc function shape with its first zero at *1/T*, where T is the symbol time.

4.2 Paging FSK Modulations

The following table estimates B/2W for various FLEX and POCSAG modulations:

Table 4-1 Modulation indexes for FLEX & POCSAG

Modulation	W (Hz)	Modulation index (fd/W)	Pass-band width (B)	B/2W
1600 binary	3200	1.5	24000	3.75
3200 binary	4800	1	28800	3
3200 4-level	3200	1.5	24000	3.75
6400 4-level	4800	1	28800	3
512 binary	1024	4.5	13500	6.25
1200 binary	2400	2	18000	3.75
2400 binary	4800	1	27000	3

The value of W=3200 Hz is arrived at for 1600 bit/s FLEX by taking the main and second lobes of the sinc function together. The value of 4800 Hz for 3200 bit/s assumes that the Bessel filter sharply removes frequencies above the middle of the second lobe of the PSD.

Unfortunately, FM signal capture is not a strong effect in the presence of Rayleigh fading, shadowing, and simulcast-beat fades. The presence of signal fades due to any of these causes reduces noise suppression and increases the overall density of noise.

In general, in the presence of fading, shadowing, and null beating, the relationship between base-band SNR and pass-band CNR becomes a straight line with a slope of 1 on dB scales. For values of B/2W of the order used in FLEX, base-band SNR is typically 0 dB for a pass-band CNR of about 7 to 8 dB. For example, a pass-band SNR of 20 dB results from a CNR of 27 dB.

4.3 FM Click Noise

Rice defines a “click” as the transition of the quadrature noise, $X_s(t)$, through 0 when the sum of the in-phase noise, $X_c(t)$ plus signal amplitude, is negative. The reader may recall our earlier phasor diagram. We are describing a situation in which the in-phase amplitude of noise is greater than the signal, and the quadrature component of the noise changes sign. When the in-phase component of noise exceeds the signal amplitude, the resultant changes angle by 180-degrees, the additional shift of the quadrature component causes a rapid 360-degree phase change. A frequency discriminator will differentiate this rapid phase shift, creating an "FM click."

The time for the click will depend only on the bandwidth of the IF filter that feeds the discriminator. The energy released in the click will be characteristic of the 360-degree shift in the discriminator. The shape and amplitude of the click will thus be *invariant* for any given receiver. Dynamically, when the CNR becomes less than 10 dB, (FM threshold) click noise dominates the discriminator output. The duration of a click is about the inverse of the bandwidth of the IF filter[11], *B*, which comes to around 40 μs for paging modulations, as can be seen from the table above (see Table 4-1).

Whenever a fade occurs that causes the CNR to fall below threshold, click noise becomes extremely prevalent in a limiter-discriminator circuit.

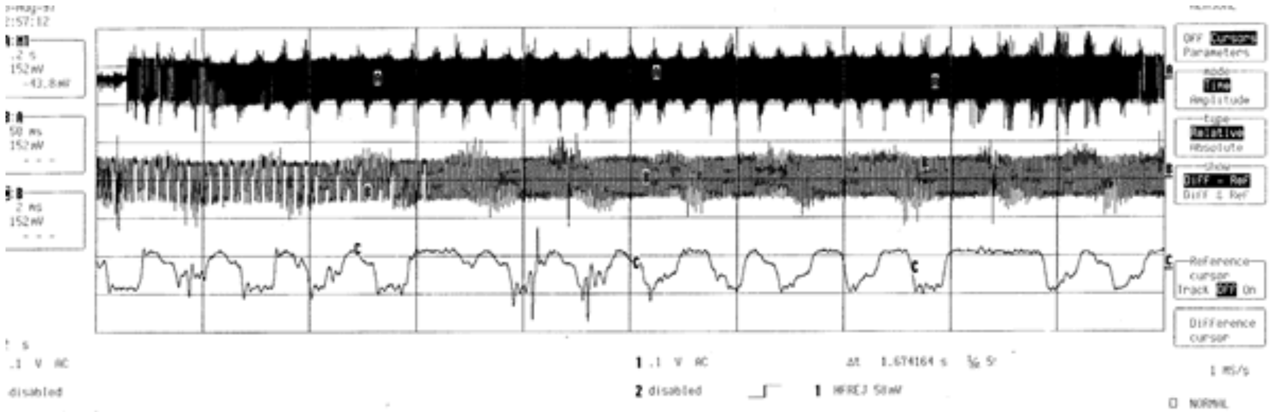
4.3.1 FM Clicks during Simulcast Fades

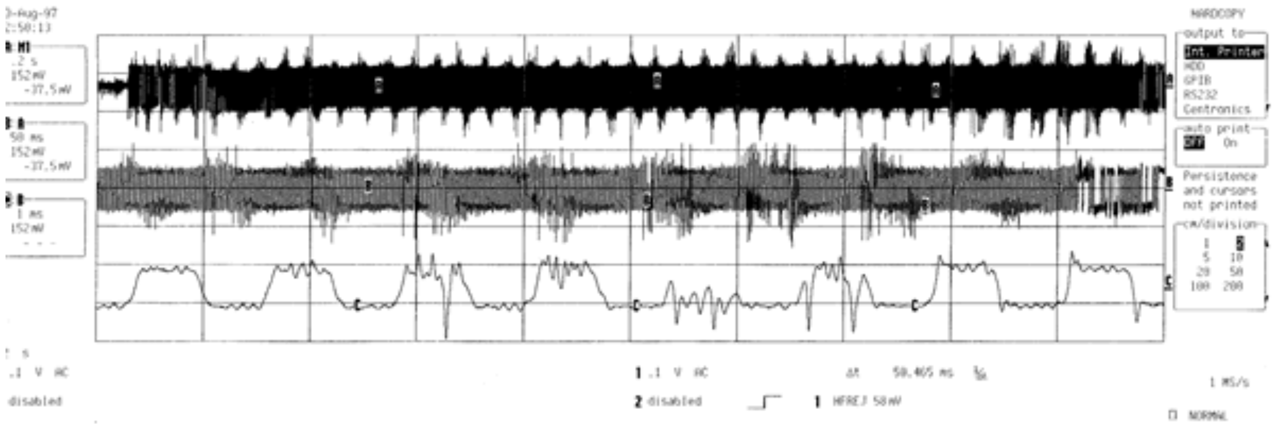
A simulcast fade is a deterministic process, in the sense that it can be predicted to occur at the beat frequency. During the fade, CNR collapses; and in a limiter-discriminator receiver, if the system falls below threshold, noise capture will occur.

The figure below is a set of oscilloscope traces showing the base-band output of a FLEX receiver going through carrier beating between two dominant transmitters. These traces were taken on the 8th floor of the PageMart offices at 6688 N. Central Expressway in Dallas.

At the longest time scales, in the upper traces, clearly show impulsive noise at the regular beat frequency of around 20 Hz, which was forced into the two dominant transmitters. The bottom traces, at the highest time resolution, show the presence of FM clicks, which appear at random with respect to the bit sequence. That is, there is no preferential time for the clicks to occur with respect to the symbol time.

Figure 4-3 Click noise due to simulcast beat fading



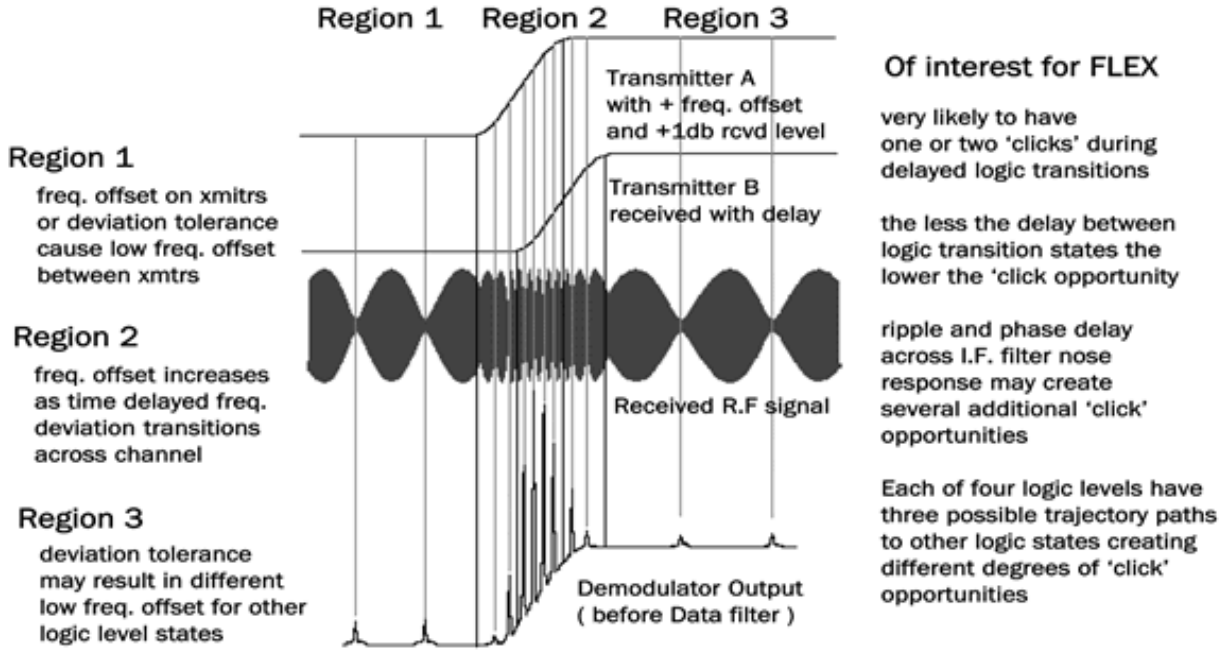


4.3.2 FM Clicks during Symbol Transitions in SDS

The following figure shows a more detailed view of the RF (or IF) signals and the resulting discriminator output during a combination of simulcast fading and during an extended symbol transition with significant SDS. In Region 1, before the transition, and Region 3, after the transition beat patterns are seen; and click noise is present in the demodulated output (at the bottom of the figure.)

Region 2 is a time of rapid phase accumulation and in this extended example, many clicks are seen during the inter-symbol transition.

Figure 4-4 FM clicks with FSK modulation & SDS



In a more typical case, there is likely to be only one click right at the inter-symbol transition. The figure also shows the phenomenon that the beat pattern in Region 1 is different than the beat pattern in Region 3. That is, the natural carrier offsets, with modulation present, will likely be different at different symbols.

5. Impact of Click Noise on Paging Receivers

Assume that any destructive interference event will push the receiver into noise capture, and that during noise capture a click is 100% probable. We have identified two processes unique to simulcast that have the opportunity to generate destructive interference; namely, simulcast beating and simulcast delay. It becomes apparent that we must consider the impact of click noise on paging FSK modulations.

In a simple binary FSK receiver (*e.g.,* POCSAG) the impact of click noise is likely to be minimal, especially where the two binary signal levels are "railed". In that case, a click will simply drive to either voltage rail. If the click is in the same direction as the bit, it will not even be observed. If it is in the opposite sense, it will "cut" a narrow slice out of the bit.

For example, at 512 bit/s, a bit time is 2,000 µs. Even a relatively long sequence of 40 µs clicks due to simulcast beat fading could have little impact on a simple "integrate-and-dump" receiver. Likewise, one or more clicks at the bit transition time would hardly be noticed in this system.

On the other hand, a 4-level FLEX receiver follows its limiter-discriminator with a PAM decision circuit. Look again at Figure 2-1, with the idea in mind that the three dotted lines are the decision levels of the PAM circuit. To ensure that the signal remains perfectly aligned with the decision levels, an automatic level control circuit must be employed that senses the extreme signal values and the "0 Volt" level. FM click noise is highly disruptive to such a circuit.

The signal (complete with clicks) cannot be railed. Once it is converted to PAM, the signal must be handled in a linear fashion. Hence, click noise is additive to the signal. This can be seen from Figure 4-3, although the figure only shows a FLEX binary signal. Click noise will preferentially impact the inner symbols of FLEX. The outer symbols have a 50% chance of a click aligning with the symbol. An inner symbol will invariably be driven into the errored state by a click[12].

Unrailed click noise will also have a disruptive impact upon another necessary circuit in a 4-level FLEX receiver: the symbol clock recovery circuit. Symbol clock is typically obtained by squaring the signal to create a timing signal that has deep notches at the symbol transition times. By passing this highly regular output through a narrow-band filter, a symbol clock can be obtained[13]. Click noise will pass through the squaring circuit looking like inter-symbol notches, but will appear at the wrong times, causing clock jitter.

In general then, the reception of FLEX at 6400 bit/s with a limiter-discriminator circuit will be highly impacted by the two processes of SBF and SDS that can create destructive interference fades. Let us consider some distinctions between the two.

In SBF, a regular pattern of fades is present. During long fades, significant bursts of click noise may occur. It is desirable to avoid these long bursts and the natural way to do this is by forcing the fades to be as short as possible.

In SDS, the destructive fades are localized at the inter-symbol transitions. The likelihood of a fade is related to the maximum SDS between received signals. Above, we estimated that for FLEX at 6400 bit/s, a T_m of 250 µs would guarantee a fade at each inter-symbol transition. Assume the worst case: that a fade yields a click with 100% probability and that a click yields a bit error with 100% probability[14]. This suggests that for a raw BER of 10%, the T_m should be less than 25 µs in any over-lap region where two or more transmitters have about equal value. A more pragmatic value may be around 40 to 50 µs, assuming that SDS clicks are about 50% effective in causing bit errors.

Another factor worth noting is that most practical situations will involve more than two transmitters in the generation of SDS. The case just outlined involved only two simulcast signals. Recall that Hess' Tm metric provides a way to convert a complex situation of multiple signals at different powers and delays to a reference model with two equal power signals at the effective offset time of T_m. This suggests a hypothesis, *which should be verified*, that the single value of Tm is all that is needed to estimate SDS click noise.

Another observation is that the value of T_m at around 50 µs suggests a T_{rms} of 25 µs, which is far less than the ISI threshold (of 80 µs) for FLEX at 6400 bit/s. **In short, SDS click noise yields significant BERs at a lower value of T_m than linear ISI does**.

One last observation is that ReFLEX25 pagers have been observed to have fewer problems with SDS than FLEX, with both operating at 6400 bit/s. An initial explanation for this was that ReFLEX25 devices had equalization for ISI. This analysis suggests a much simpler explanation; namely, that ReFLEX25 has ½ the deviation between symbols as FLEX. Hence, at the same symbol rates and SDS, ReFLEX25 devices will accumulate ½ the phase, and have ½ the likelihood of generating clicks. Thus, ReFLEX25 devices at 6400 bit/s (4-level) will perform in an SDS environment like FLEX at 3200 bit/s (4-level.) Another possible explanation is offered below (see 6.4.)

6. Recommendations

6.1 Probability distributions of fast & slow fading

In 3.5, we described the possible impact of simulcast interference on the Log-normal probability distribution function of received signal strength. We also explained how important that distribution was to the estimation of fade margins for reliable signaling. **An obvious recommendation is to perform a sequence of experiments, as simulations, lab tests, and field tests, to test the hypothesis that simulcast interference changes the PDF of signal strength.** Recall also the earlier speculation that there may be an interaction between any change in the PDF of slow (Log-normal) fading with the fast fading PDF being Ricean as opposed to Rayleigh.

Any verification of this effect could be fed back into the design of our RF simulations tools to provide improved predictions of paging reliability.

6.2 Avoidance of Long Simulcast Fades

The natural method to avoid long intervals in a simulcast fade is to increase beat frequencies. This is done with an engineered frequency offset plan. Selwyn Hill has already worked hard to demonstrate the effectiveness of frequency offsets in improving paging reliability for FLEX. This report does not propose to repeat his work; the interested reader can obtain Hill's documents for review. However, the following two graphs are included as a demonstration of the benefits of offset plans.

The two graphs show paging reliability for A-phase and B-phase FLEX pagers, using Glenayre or Motorola transmitters, for a range of offsets. A number of observations can be made.

First, A-phase pagers are generally superior in performance to B-phase pagers. The reasons for this are given in a separate report[15].

Second, paging reliability is considerably improved with the use of offsets.

Third, Glenayre transmitters outperform Motorola transmitters, especially for B-phase pagers.

Finally, the effect of simulcast beat fading on reliability is indicative of an "irreducible" error rate; that is, signal strength is good in these tests (-75 dBm.) The impact of the offset plans are significant: from under 70% reliability for a Nucleus transmitter with no offsets to over 90% with a 16 Hz offset.

The clear recommendation is to utilize frequency offset plans for FLEX at 6400 bit/s wherever a market shows pager use in simulcast environments with Ricean (as opposed to Rayleigh) fading statistics. For example, a flat suburban market with two or three transmitters may have no significant SBF; and hence, no need for offsets. A dense urban environment with ten transmitters surrounding numerous office towers may be a trouble area where an offset plan is essential.

Figure 6-1 Paging Reliability for A-phase FLEX at various frequency offsets.

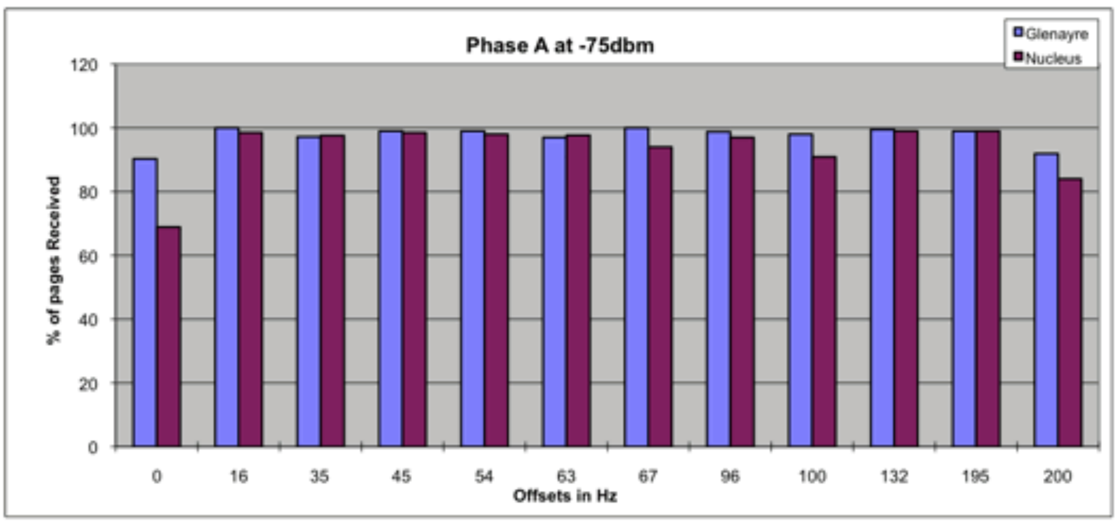
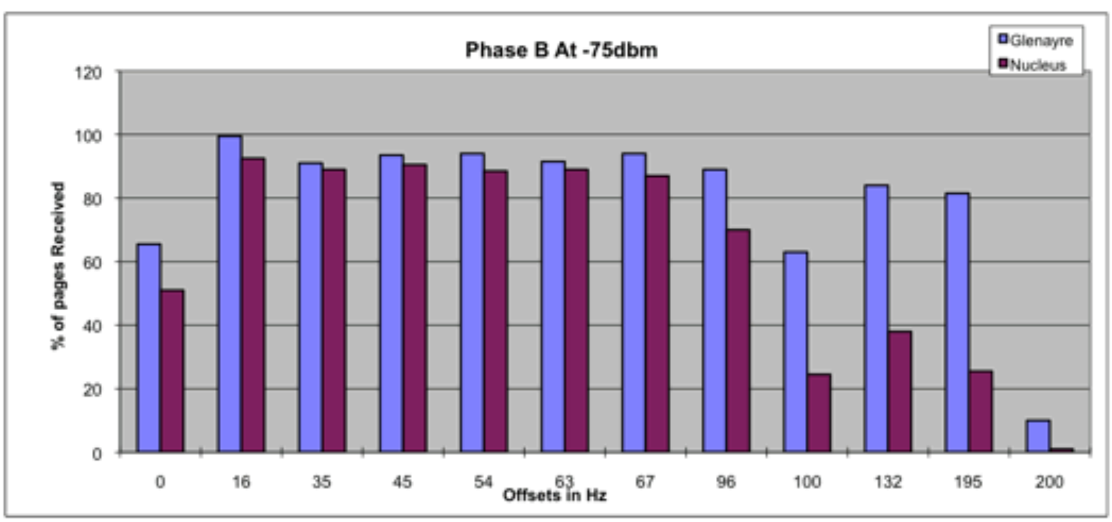


Figure 6-2 Paging Reliability for A-phase FLEX at various frequency offsets.



6.3 Avoidance of SDS

Previous analysis of the impact of SDS on FSK signaling have focused on the issue of inter-symbol interference. This report suggests that the FM click noise associated with SDS is a more significant effect. Like our analysis of simulcast beating, our analysis of SDS assumed stable amplitudes for the received signals. This again suggests that FM click noise could be most profound in Ricean (not Rayleigh) fading environments; for example, office towers. However, unlike simulcast beat patterns, symbol transitions are short with respect to Rayleigh or Ricean fade processes. But while signal amplitudes may be stable during a symbol transition, they may change strongly between symbol transitions in a Rayleigh environment.

In fact, in an office tower, near-by transmitters may provide signals that have Ricean statistics while more distant transmitters continue with Rayleigh statistics. Hence, SDS effects may "come and go" as signals fade. Also, even in an office tower slow, Log-normal shadowing is also important, since building cores, walls and other obstructions can still obstruct signal paths.

Since SDS FM click noise can be translated directly to an impact on BER, and the probability of a click can be related to accumulated phase angle, which depends only on T_m, a algorithm to estimate SDS presents itself.

For regions of interest in a market, designate which sites have Ricean and which have Rayleigh statistics. For each location of interest, repeat the following Markov analysis to convergence to a mean. Compute Tm for the signals subject to Log-normal shadowing and Rayleigh or Ricean fading, as appropriate. Repeat to find the average Tm at the location. From the T_m, calculate a BER. This algorithm generates a map of irreducible BER due to SDS clicks. The algorithm can be "tuned" by changing the likelihood that clicks cause bit errors.

In earlier work, we have studied in some detail the PDFs of T_m under various situations. During this work, we discovered that these PDFs are highly irregular. As a consequence, any algorithm to fully characterize the PDF at all sites in a market was costly of compute-time. The method described above avoids this by concentrating only on convergence to the mean of T_m. Since T_m may be used as a proxy for BER due to SDS clicks, this approach may yield useful maps.

At present, PageMart is using a simulated annealing method to minimize SDS in critical areas of its markets. Simulated annealing has proven highly effective in non-linear optimization problems such as this. By applying simulated annealing to the minimization of mean Tm as an objective function, a reasonably tractable approach may be achieved to the optimization of systems for FLEX at 6400 bit/s.

The recommendation of this report is to continue to investigate and fine-tune our RF analysis and optimization tools in this area.

6.4 Optimal Receiver Design

Finally, it is worth noting that the generation of click noise is unique to the limiter-discriminator form of FM detection. That is, there are other well-known designs for FM/FSK detection that avoid or ameliorate clicks.

The whole topic of FM click noise is well-known also. In fact, the first FM land mobile voice system, for a police service in the north-eastern US in the 1930s, was severely influenced by FM click noise. As a consequence of the analysis of this service, the practice of FM pre-emphasis and de-emphasis filtering was adopted. The de-emphasis filter is a wide-band integrator in the receiver, which smoothes out clicks. To compensate for the receiver's de-emphasis, the modulating signal is differentiated, or pre-emphasized. In effect, this turns FM into PM, and clicks become "steps." These step-offsets are of little impact in an "ac" coupled audio signal.

Among the FM detector designs that may be used for FSK data and that avoid clicks is the phase-locked loop (PLL). A typical PLL decoder will still be front-ended with a limiter, and so noise bursts are still feasible. However, the PLL has superior signal capture performance in high noise input situations than the discriminator does.

Another design that completely avoids clicks is the optimal matched filter decoder. In this design, a bank of four band-pass filters accumulate symbol energy for each symbol time. At the end of the symbol time, a decision circuit selects the symbol associated with the highest energy. There is no limiter (although there may be slow AGC) and no differentiation. There is no click noise at all. The base station receivers for ReFLEX25 all use this method. While this design may appear expensive for a simple (numeric) pager, it has the singular advantage of avoid all of the click noise effects described in this report.

The recommendation of this section is to share the information of this report with our FLEX and ReFLEX25 device vendors and to actively encourage them to investigate alternative receiver designs that avoid high BERs due to click noise.

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[1] It is possible, with sufficient gain, to construct a paging receiver at "zero-IF" using quadrature digital signal processing methods. Philips built such receivers for POCSAG over a decade ago.
[2] The two gaussian distributions are just the in-phase and quadrature components of the received signal from all scattering sources.
[3] Most inexpensive paging receivers are carrier asynchronous.
[4] The vector wave number, **k**, is 2πr/λ , where **r** is a unit vector in the direction of the radio wave and λ is its wavelength.
[5] A Dirac delta in the time domain instead of a Dirac delta in the frequency domain.
[6] In theory, the 4th power over plane earth. In practice, more like 3.6-3.8 at UHF.
[7] 18 dB is 3σ.
[8] Now 18 dB is only 1.5σ.
[9] Looked at another way, the τ_{rms} for 2 equal-power signals is just ½ their time offset.
[10] Proakis, John, G., *Digital Communications,* McGraw-Hill, 1983, pp.124-135.
[11] Basically, a "click" is the impulse response of that filter.
[12] Recall the Gray coding of FLEX.
[13] There is a bit more to it, but those details are unimportant here.
[14] This is an upper bound on errors.
[15] A. Angus, *Error processes in FLEX & ReFLEX25,* PageMart Confidential report.
